Optimality of the Bottleneck Product Rate Variation Problem with Batching

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Abstract

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- Bottleneck Product Rate variation Problem with Batching is Formulated.

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- A Pareto Optimal Solution is Proposed.

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- Bottleneck Product Rate variation Problem with Batching is Formulated.
- A Pareto Optimal Solution is Proposed.
- A Relation between Optimal Sequences is Presented.

Motivation

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NOKIA NEPAL Receives Following Demands

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$Model_A = 1000$, $Model_B = 600$, $Model_C = 400$.

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Production manager requests to Cheif consultant.

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NOKIA NEPAL Receives Following Demands

$Model_A = 1000$, $Model_B = 600$, $Model_C = 400$.

Production manager requests to Cheif consultant.

Dr. Dhamala, could you suggest the best production sequence of the demands ?

Motivation

Dhamala is busy in computer with his crew.

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After some times...

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After some times...

Dhamala answers to Production manager.

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Mr. Tamang, here is the best sequence...

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(ABCAABACBA)

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After some times...

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(ABCAABACBA)(ABCAABACBA) . . .

200 times.

HOW ?

Mixed-model Just-in-time Multi-level Production System

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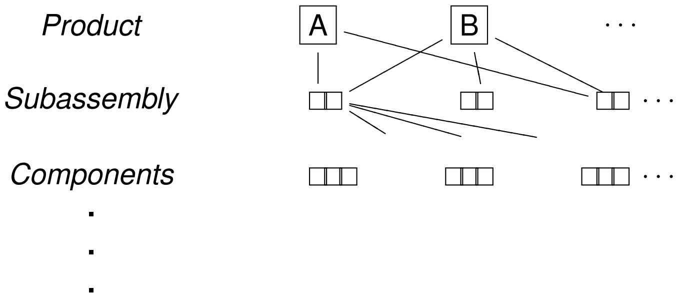

Usage rate of parts or models:

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as constant as possible.

How to keep usage rate of parts as constant as possible?

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Output Rate Variation Problem.

Output Rate Variation problem yields

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Production sequence in each level.

BUT

Output Rate Variation problem is NP-hard.

How to keep usage rate of models as constant as possible?

How to keep usage rate of models as constant as possible?

Product Rate Variation Problem.

Product Rate Variation Problem is solvable.

Product Rate Variation Problem is solvable.
Pseudo-polynomial solution method exists.

Example

Example

(ABCAABACBA)...

Example

(ABCAABACBA)...

Sequence with no switch-over cost.

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(A

Example

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(A □

Example

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(A □ B □ C □ AA □ B □ A □ C □ B □ A)...

Example

(ABCAABACBA)...

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Sequence with switch-over cost.

Solution of Product Rate Variation Problem with Batching

Solution of Product Rate Variation Problem with Batching

(A □ B □ C □ AA □ B □ A □ C □ B □ A)...

Batch Number and Size

$$\textbf{maximize} \quad \frac{\tilde{d}_0}{\mathcal{D}}$$

Batch Number and Size

maximize $\dfrac{\tilde{d}_0}{\mathcal{D}}$

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maximize $\frac{\tilde{d}_0}{\mathcal{D}}$

maximize $\tilde{d}_0$

maximize $\mathcal{D}$

Model determines the batch number and batch size.

Sequence of Batches

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minimize

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$$max_{i,k}(\gamma_i |\tilde{x}_{ik} - k\tilde{r}_i|)^m$$

Sequence of Batches

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Model determines sequence of the batches.

Determination of Batch Size

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Enumeration Method

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- Find the largest possible number of batches.

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Enumeration Method

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- Enumerate all possible number of batches with $n \leq \mathcal{D} \leq \mathcal{D}_{max}$.
- Calculate number of batches of model i.

$$\tilde{d}_i = \left\lceil \frac{d_i}{\left\lfloor \frac{\frac{T}{\mathcal{D}} - s_i}{p_i} \right\rfloor} \right\rceil.$$

Determination of Batch Sequence

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Perfect Matching yields Feasible Solution.

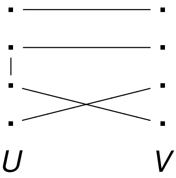
Determination of Batch Sequence

Perfect Matching yields Feasible Solution.

Bisection search yields Optimal Solution.

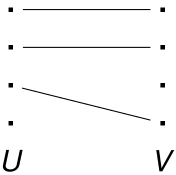
Matching

Matching

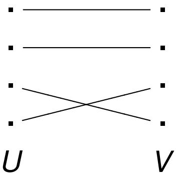


Bipartite Matching

Bipartite Matching

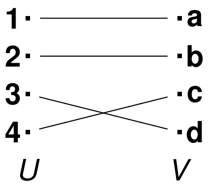


Perfect Bipartite Matching

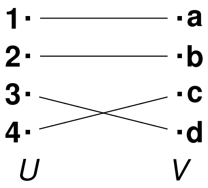


Sequence from Perfect Bipartite Matching

Sequence from Perfect Bipartite Matching



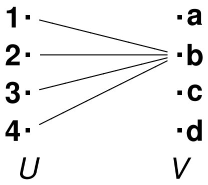
Sequence from Perfect Bipartite Matching



abdc

U-Convex Bipartite Graph

U-Convex Bipartite Graph



Perfect matching

Sequencing Times

Perfect matching

Sequencing Times

Earliest sequencing time.

$$E_m(i,j) = \left\lceil \frac{j - \frac{1}{\gamma_i}\sqrt[m]{B}}{\tilde{r}_i} \right\rceil$$

.

Perfect matching

Sequencing Times

Latest sequencing time.

$$L_m(i,j) = \left\lfloor \frac{j - 1 + \frac{1}{\gamma_i} \sqrt[m]{B}}{\tilde{r}_i} + 1 \right\rfloor$$

.

Perfect matching

Sequencing Times

Latest sequencing time.

$$L_m(i,j) = \left\lfloor \frac{j - 1 + \frac{1}{\gamma_i} \sqrt[m]{B}}{\tilde{r}_i} + 1 \right\rfloor$$

.

Sequencing times can be calculated in $O(D)$ time

.

Perfect matching

Theorem

Perfect matching

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If (i, j) be sequenced within $[E_m(i, j), L_m(i, j)]$, then the level curves do not exceed B.

Perfect matching

Theorem

If (i, j) be sequenced within $[E_m(i, j), L_m(i, j)]$, then the level curves do not exceed B.

$$(\gamma_i \, |j - k\tilde{r}_i|)^m \leq B.$$

B - Bottleneck.

Feasible Solution

Feasible Solution

Order preserving perfect matching $\Leftrightarrow$ Feasible solution.

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Theorem: Existence of a perfect matching

Feasible Solution

Order preserving perfect matching $\Leftrightarrow$ Feasible solution.

Theorem: Existence of a perfect matching

Let K be either an interval $[k_1, k_2]$ in $\tilde{V}_1$ or the neighborhood of $[k_1, k_2]$ in $\tilde{V}_1$ for all $k_1, k_2 \in \tilde{V}_1$ with $k_1 \le k_2$ and $\left[\tilde{E}_m(i,j), \tilde{L}_m(i,j)\right] \cap [k_1, k_2] \neq \phi$, Hall's condition is equivalent to the inequalies:

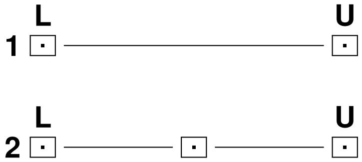
$\sum_{i=1}^{n}(\left\lfloor k_2\tilde{r}_i + \frac{1}{\gamma_i}\sqrt[m]{B}\right\rfloor - \left\lceil (k_1 - 1)\tilde{r}_i - \frac{1}{\gamma_i}\sqrt[m]{B}\right\rceil) \geq k_2 - k_1 + 1$ and

$\sum_{i=1}^{n}(\left\lceil k_2\tilde{r}_i - \frac{1}{\gamma_i}\sqrt[m]{B}\right\rceil - \left\lfloor (k_1 - 1)\tilde{r}_i + \frac{1}{\gamma_i}\sqrt[m]{B}\right\rfloor) \leq k_2 - k_1 + 1.$

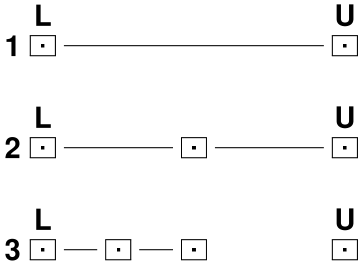
Bisection Search

Bisection Search

Bisection Search



Bisection Search



Lower and Upper Bottlenecks

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Lower Bottleneck

$$L = (\gamma_i(1 - \tilde{r}_{max}))^m$$

Lower and Upper Bottlenecks

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Upper Bottleneck

$$U = (\gamma_i(1 - \frac{1}{\mathcal{D}}))^m$$

Lower and Upper Bottlenecks

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$$L = (\gamma_i(1 - \tilde{r}_{max}))^m$$

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Theorem

Lower and Upper Bottlenecks

Lower Bottleneck

$$L = (\gamma_i(1 - \tilde{r}_{max}))^m$$

Upper Bottleneck

$$U = (\gamma_i(1 - \frac{1}{\mathcal{D}}))^m$$

Theorem

Bisection search determines a minimum B in $O(logD)$ time.

Relation between optimal sequence

Relation between optimal sequence

Theorem

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Theorem

If s is feasible to Problem F_1, then s is feasible to Problem F_m.

Relation between optimal sequence

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If s is feasible to Problem F_1, then s is feasible to Problem F_m.

- The theorem is not true for optimality.

Relation between optimal sequence

Theorem

If s is feasible to Problem F_1, then s is feasible to Problem F_m.

- The theorem is not true for optimality.
- Converse is not true for both feasible and optimal cases.

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ACKNOWLEDGEMENT

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TANKA NATH DHAMALA

HAPPY 4^{th} ANNUAL DAY

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Thank You.