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Purchasing Inventory Models for Non-Deteriorative items with Constant, Linear and Quadratic Demand – A Comparative Study

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Abstract

In this paper, a purchasing inventory models for non-deteriorative items with constant, linear and quadratic demand is considered and the optimum solution is derived in higher order equation. Three models are developed. In the first model, the purchasing inventory model with constant demand, in the second model, linear demand and in the third model, quadratic demand for non-deteriorative items is considered. A mathematical model is developed for each model and the optimal production lot size which minimizes the total cost is derived. The optimal solution is derived and an illustrative example is provided. The validation of result in this model was coded in Microsoft Visual Basic 6.0.

Key words: Inventory, Demand, constant, linear, quadratic.

1. Introduction

In general, the classical inventory models assume constant demand over an infinite planning horizon. This assumption is valid during the maturity phase of the product life cycle and for a finite period of time. In other phases of a product life cycle demand for the product may increase after its successful in introduction into the market or decrease due to, for example, introduction of new competitors products. Several papers were published with constant, linearly increasing or decreasing demand and few papers were published with quadratic demand. Also, most works on inventory models do not take the optimal solution in higher order equation. Since the energy crises in the mid 1970s, many countries have experienced high annual inflation rates and a consequently sharp declines in the purchasing power of money. Because inventory systems need to invest amounts of capital to purchase inventories, the effect of inflation and time-value of money cannot be ignored when we determine the optimal inventory policy. Economic order quantity (EOQ) models have been studied since Harries [1] presented the famous EOQ formulae. Dave and Patel [2] considered an EOQ model in which the demand rate is changing linearly with time and the deteriorating is assumed to be a constant fraction of the on hand inventory. Chandra and Bahner [3] extended the results in Misra to allow for shortages. Vrat and Padmanabhan [4] developed an inventory model under a constant inflation rate for initial stock-dependent consumption rate. Xu and Wang [5] presented an EOQ model for exponentially deteriorating items with linearly time varying demand and finite shortages cost. Datta and Pan [6] developed a model with linear time dependent rates and shortages to investigate the effect of inflation and time-value of money on a finite horizon policy. Saker and Pan [7] assumed a finite replenishment model and analyzed the effects of inflation and time-value of money on order quantity when shortages are allowed. Chen [16] proposed a generalized dynamic programming model for inventory items with weibull distributed deterioration and the demand rate is assumed to be time-proportional, shortages are allowed and completely backordered and the effects of inflation and time-value of money are taken into consideration. Wee and Law [10] developed a deteriorating inventory model taking into account the time-value of money for a deterministic inventory system with price-dependent demand. Chung and Tsai [9] derived an inventory model for deteriorating items with the demand of linear trend and shortages during the finite planning horizon considering the time value of money. Skouri and Papachristos [11] developed an inventory model for the deterioration of items occurs at a fixed rate independent of time. The model allows for partially backlogging and the backlogging rate is an exponentially decreasing, time-dependent function specified by a parameter. Ghosh and Chaudhuri [12] developed an inventory model for a deteriorating item, a quadratic time-vary demand and shortages in inventory. A two-parameter weibull distribution is taken to represent the time of deterioration. Ghosh and Chaudhuri [13] developed an EOQ model over a finite time-horizon for a deteriorating item with a quadratic, time-dependent demand, allowing shortages in inventory and the rate of deterioration is taken to be time-proportional and it is assumed that shortage occur in every cycle. Mandal [14] considered in which it is depleted not only by demand but also by deterioration. The Weibull distribution, which is capable of representing constant, increasing and decreasing rate of deterioration, is used to represent the distribution of the time to deterioration. Mahata [15] constructed an inventory level for deteriorating items with instantaneous replenishment, exponential decay rate and a time varying linear demand without shortages under permissible delay in payments. Chauduri et al. [17] an economic order quantity inventory problem is discussed over a finite time for deteriorating items with shortages, where the demand rate is of the ramp-type. It is assumed that a constant fraction of the on-hand inventory deteriorates per unit time and time value of money and the effects of inflation are taken into account. Cheng et al. [16] considered an inventory model for time-dependent deteriorating items with trapezoidal type demand rate and partial backlogging. Ghosh et al. [18] considered an optimal inventory replenishment policy for a deteriorating items time-quadratic demand and time-dependent partial backlogging which depends on the length of the waiting time for the next replenishment over a finite time horizon and variable replenishment cycle. Mishra et al. [19] considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deteriorating is time proportional and the model considered here allows shortages and the demand is partially backlogged. Hsu and Hsu [20] developed inventory model for vendor-buyer coordination under an imperfect production process and the proportion of defective items in each production lot is assumed to be stochastic and follows a known probability density function. Singh and Singh [21] developed an inventory model with power form stock-dependent demand rate and the demand rate is assumed to be a polynomial form of current inventory levels in own warehouses and also it is assumed that retailer first fulfills the demand directly from the RW until the inventory level in the RW reaches to the zero level after that, demand is fulfilled from OW. Kumar and Singh [22] an EOQ model with two level of storage is developed when units in inventory deteriorate at a constant rate and demand is stock dependent. The salvage value is allied to deteriorated units. When stock level exceeds the capability of own warehouse (OW), it is to be kept in additional rented warehouse (RW) with higher holding cost than OW. The inventory held in RW is transferred to OW in continuous release pattern till the stock in RW gets exhausted. Sharma [23] considered the production rate as demand dependent, which is more realistic in our general life. The planning horizon is infinite. The time dependent rate of deterioration is taken into consideration and demand rate is price dependent. We consider that shortages are allowed and partially backlogged. The demand rate is regularly assumed to be either constant or time-dependent, but independent of the stock levels. Since the demand rate is not only influenced by stock level, but also is associated with the selling price. Mondal and Maiti [24] for defective items, realistic optimal production-inventory models with fuzzy time period have been formulated and solved. Here, the rate of production is assumed to be a function of time and considered as a control variable. Also, the demand is time dependent and known. The values of an integral over a fuzzy interval are obtained with the help of Zimmerman's technique and fuzzy Riemann integral theory. In this paper, a purchasing inventory models for non-deteriorative items with constant, linear and quadratic demand is considered and the optimum solution is derived in higher order equation. Three models are developed. In the first model, the purchasing inventory model with constant demand, in the second model, linear demand and in the third model, quadratic demand for non-deteriorative items is considered. A mathematical model is developed for each case and the optimal production lot size which minimizes the total cost is derived. The optimal solution is derived and an illustrative example is provided. The validation of result in this model was coded in Microsoft Visual Basic 6.0. The rest of the paper is organized as follows: Section 2 presents the assumptions and notations. Section 3 is for mathematical formulation, numerical examples and a comparative study is carried out. Finally, this paper summarizes and concludes in section 4.

2. Assumptions and Notations

2.1 Assumptions: The following assumptions are used to formulate the problem.

1) The inventory system involves only one item.

2) Lead time is zero. There is sufficient capacity and capital to procure the desired lot size.

3) Shortages are not allowed

4) The demand is constant per unit/ year (D), linear ($a+bt$) and quadratic function ($a+bt+ct^2$) where a>0, $b \neq 0, c \neq 0$, at time t and it is continuous function of time. Here, a,b and c are constants and "a" stands for the initial demand "b" and "c" are positive trend in demand.

5) There is no repair or replacement of the deteriorated items.

6) The replenishment occurs instantaneously at an infinite rate.

2.2 Notations: The following notations are used in our analysis.

1. D – Demand rate in units per unit time
2. Q^*-Optimal size of production run
3. C_h - Inventory holding cost per unit/unit time.
4. C_0 – Ordering cost of inventory per order
5. T – Cycle time
6. TC - Total cost

3. Purchasing Inventory Models for non-deteriorative items with Constant, Linear and Quadratic Demand:

In most of the inventory model, demand has considered as a constant. But in realistic situation, demand is always not a constant function. It is varying according to time. So, this model developed a deterministic inventory model in which demand is constant The demand is constant (D), linear ($a+bt$) and quadratic function ($a+bt+ct^2$) where a>0, $b \neq 0, c \neq 0$, at time t and it is continuous function of time. Here, a,b and c are constants and "a" stands for the initial demand "b" and "c" are positive trend in demand. A few papers were published in quadratic demand. But, in this model the optimal solution is developed in third and fourth order equations.

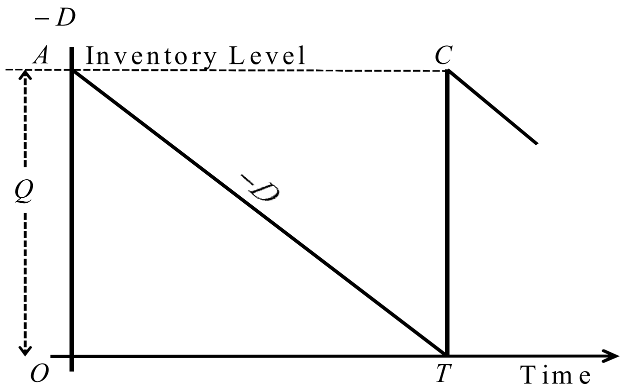

Figure 1: Purchasing Inventory Model

3. 1 Constant Demand: It assumes that demand rate for the item is constant and known with uncertainty. During the stage (0,T), the inventory of good items increases due to purchases but decreases duet to demand. Thus, the inventory differential equation is

$$\frac{d}{dt}I(t) = -D \tag{1}$$

with the boundary conditions, I(0) = Q and I(T) =0 (2)

From (1), I(t) = D(T-t) (3)

Total cost: Total cost consists of ordering cost and holding cost

i) Ordering cost = $\dfrac{C_0}{T}$ (4)

ii) Holding cost = $\dfrac{C_h}{T}\int_0^T I(t)dt = \dfrac{C_h}{T}\int_0^T D(T-t)dt = \dfrac{DTC_h}{2}$ (5)

Total cost (TC) = $\dfrac{C_0}{T} + \dfrac{DTC_h}{2}$ (6)

Optimality: It can be easily shown that TC (T) is a convex function in T. Hence, an optimal cycle time T can be calculated from

$$\frac{d}{dT}TC(T)=0 \text{ and } \frac{d^2}{dT^2}TC(T)>0$$

Differentiate the equation (6) with respect to T,

$$\frac{d}{dt}(TC)=\frac{-C_0}{T^2}+\frac{DC_h}{2}$$

and $$\frac{d^2}{dt^2}(TC)=\frac{2C_0}{T^3}>0 \qquad \text{Therefore, } T=\sqrt{\frac{2C_0}{DC_h}} \qquad (7)$$

Numerical Example: Let us consider the cost parameters D = 4500, $C_h = 10$, $C_0 = 100$

Optimum Solution: T = 0.0666, Q = 300, Setup cost = 1500, Holding cost = 1500,

Total cost = 3000

Sensitivity Analysis

The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

Table 1: Effect of Demand and cost parameters on optimal values

Parameters		Optimum values				
		T	Q	Setup cost	Holding cost	Total cost
C_0	80	0.0597	268.33	1341.64	1341.64	2683.28
	90	0.0632	284.60	1423.02	1324.02	2846.05
	100	0.0666	300.00	1500.00	1500.00	3000.00
	110	0.0699	314.64	1573.21	1573.21	3146.43
	120	0.0730	328.63	1643.17	1643.17	3286.33
C_h	8	0.0745	335.41	1341.64	1341.64	2683.28
	9	0.0703	316.23	1423.02	1423.02	2846.05
	10	0.0666	300.00	1500.00	1500.00	3000.00
	11	0.0635	286.04	1573.21	1573.21	3146.43
	12	0.0608	273.86	1643.16	1643.16	3286.33

Observations: From the table 1, it is observed that

1. With the increase in purchasing cost per unit (C_0), optimum quantity (Q*), Setup cost(C_0), Holding cost (C_h), cycle time (T) and total cost increases.

2. With the increase in holding cost per unit (C_h), optimum quantity (Q*), set up cost and holding cost and cycle time (T) decreases but total cost increases.

3.2 Linear Demand: In general, the classical EOQ models assume constant demand over an infinite planning horizon. This assumption is valid during the maturity phase of the product life cycle and for a finite period of time. In other phases of a product life cycle demand for the product may increase after its successful introduction into the market or decrease due to, for example, introduction of new competitive products. The inventory model with a linear trend in demand was initially introduced by Donaldon. Several other papers were also published with linearly increasing or decreasing demand. But, in this model, the optimal solution is derived in third order equation.

The differential equation is

$$\frac{d}{dt} I(t) = -(a + bt); \quad 0 < t < T \tag{9}$$

With the boundary conditions I(0) = Q and I(T) = 0

The solution of (8), $I(t) = a(T - t) + \frac{b}{2}(T^2 - t^2)$ (10)

Total cost : Total cost consists of setup cost and holding cost .

i) Setup cost = $\frac{C_0}{T}$ (11)

ii) Holding cost = $\frac{C_h}{T}\left[\int_o^T a(T - t) + \frac{b}{2}(T^2 - t^2)\right]dt$

$$= \frac{C_h}{T}\left[\frac{a}{2}T^2 + \frac{b}{3}T^3\right] = \frac{C_h}{6}\left[\frac{a}{2}T + \frac{b}{3}T^2\right]$$

$$= \frac{C_h}{6}\left[3aT + 2bT^2\right] = \frac{C_h}{6}\left[3aT + 2bT^2\right] \tag{12}$$

Total Cost (TC) = $\frac{C_0}{T} + \frac{C_h}{6}\left(3aT + 2bT^2\right)$ (13)

Optimality: It can be easily shown that TC (T) is a convex function in T. Hence, an optimal cycle time T can be calculated from

$$\frac{d}{dT}TC(T) = 0 \text{ and } \frac{d^2}{dT^2}TC(T) > 0$$

Differentiate the equation (13) with respect to T,

$$\frac{d}{dt}(TC) = \frac{-C_0}{T^2} + \frac{C_h}{6}(3a + 4bT) = 0 \text{ and } \frac{d^2}{dt^2}(TC) = \frac{2C_0}{T^3} + \frac{4bC_h}{6} > 0$$

On simplification,

$$4bT^3 + 3aT^2 - \frac{6C_0}{C_h} = 0 \tag{14}$$

which is the optimum solution for cycle time in third order equation.

Numerical Example,

Let D=4500, $C_0 = 100$, $C_h = 10$, a=4250, b=3790,

$$15160T^3 + 12750T^2 - 60 = 0$$

The roots are (+) 0.06605, (-) 0.83536, (-) 0.0717 in which the positive root 0.06605 is considered for this research.

Optimum solution

T = 0.0660, Q = 297.27, Setup cost = 1513.90, Holding cost = 1458.78,Total cost =2972.68

Sensitivity Analysis

Table 2: Effect of Demand and cost parameters on optimal values

Parameters		Optimum values				
		T	**Q**	**Setup cost**	**Holding cost**	**Total cost**
C_0	80	0.0593	265.36	1349.02	1304.59	2653.62
	90	0.0627	281.74	1433.61	1383.83	2817.44
	100	0.0661	297.26	1513.90	1458.78	2972.68
	110	0.0691	312.02	1590.51	1530.08	3120.59
	120	0.0721	326.21	1663.93	1598.22	3262.15
C_h	8	0.0735	333.08	1359.65	1304.98	2664.64
	9	0.0694	313.66	1438.93	1384.02	2822.96
	10	0.0661	297.26	1513.90	1458.78	2972.68
	11	0.0631	283.19	1585.19	1529.89	3115.08
	12	0.0605	270.93	1653.30	1597.83	3251.13
a	4050	0.0675	290.59	1481.73	1424.19	2905.92
	4150	0.0667	293.95	1497.89	1441.59	2939.48
	4250	0.0661	297.26	1513.90	1458.78	2972.68
	4350	0.0654	300.55	1529.76	1475.78	3005.54
	4450	0.0647	303.80	1545.47	1492.58	3038.05
b	3590	0.0662	296.98	1511.09	1458.68	2969.77
	3690	0.0661	297.12	1512.50	1458.73	2971.22
	3790	0.0661	297.26	1513.90	1458.78	2972.68
	3890	0.0659	297.41	1515.30	1458.83	2974.13
	3990	0.0659	297.56	1516.70	1458.88	2975.58

Observations: From the table 2, it is observed that

1. With the increase in setup cost per unit (C_0), optimum quantity (Q*), Cycle time (T) , Setup cost , Holding cost (C_h) and total cost increases.

2. With the increase in holding cost per unit (C_h), optimum quantity (Q*), Setup cost and Holding cost (C_h)and cycle time (T) decreases but total cost increases.

3. Similarly, other parameters "a" and "b" can also be observed from the table.

3.3 Quadratic Demand: In most of the inventory model, demand has considered as a constant. But in realistic situation demand is always not a constant function. It is varying according to time. So, this model developed a deterministic deteriorating inventory model in which demand is quadratic function of time that is $a + bt + ct^2$ where a>0, b$\neq 0$ and c$\neq 0$at a time t and "a" stands for the initial demand and "b" is positive trend in demand. A few papers were published in quadratic demand.

But, in this model the optimal solution is developed in fourth order equation. The differential equation is:

$$\frac{d}{dt}I(t) = -(a + bt + ct^2)\,; \quad 0 < t < T \tag{15}$$

with the boundary conditions I(0) = Q and I(T) = 0 (16)

The solution of (15) is , $I(t) = a(T-t) + \frac{b}{2}(T^2 - t^2) + \frac{c}{3}\left(T^3 - t^3\right)$ (17)

Total cost : Total cost consists of setup cost and holding cost .

i) Setup cost = $\frac{C_0}{T}$ (18)

ii) Holding cost = $\frac{C_h}{T}\left[\int_o^T a(T-t) + \frac{b}{2}(T^2 - t^2) + \frac{c}{3}(T^3 - t^3)\right]dt$

On simplification,

$$\text{HC} = \frac{C_h}{T}\left[\frac{a}{2}T^2 + \frac{b}{3}T^3 + \frac{c}{4}T^4\right] = \frac{C_h}{12}\left[6aT + 4bT^2 + 3cT^4\right]$$

$$= \frac{C_h}{12}\left[6aT + 4bT^2 + 3cT^4\right] \tag{19}$$

$$\text{Total Cost (TC)} = \frac{C_0}{T} + \frac{C_h}{12}\left[6aT + 4bT^2 + 3cT^4\right] \tag{20}$$

Optimality: It can be easily shown that TC (T) is a convex function in T. Hence, an optimal cycle time T can be calculated from

$$\frac{d}{dT}TC(T) = 0 \text{ and } \frac{d^2}{dT^2}TC(T) > 0$$

Differentiate the equation (20) with respect to T,

$$\frac{d}{dt}(TC) = \frac{-C_0}{T^2} + \frac{C_h}{12}(6a + 8bT + 9cT^3) = 0$$

$$\text{and } \frac{d^2}{dt^2}(TC) = \frac{2C_0}{T^3} + \frac{C_h}{12}(8b + 36cT^{@}) > 0$$

On simplification,

$$9cT^4 + 8bT^3 + 6aT^2 - \frac{12C_0}{C_h} = 0 \tag{21}$$

which is the optimum solution for cycle time.

Numerical Example,

Let D=4500, $C_0 = 100$, $C_h = 10$, a=4250, b=2660, c= 1100

$$T^4 + 2.149T^3 + 2.575T^2 - 0.0121 = 0$$

Optimum solution

T = 0.0667, Q = 300.25, Setup cost = 1498.99, Holding cost = 1457.89, Total cost = 2956.89

Sensitivity Analysis

Table 3: Effect of Demand and cost parameters on optimal values

Parameters		Optimum values				
		T	Q	Setup cost	Holding cost	Total cost
C_0	80	0.0598	267.79	1336.88	1303.95	2640.84
	90	0.0634	284.45	1420.08	1383.06	2803.15
	100	0.0667	300.25	1498.99	1457.89	2956.89
	110	0.0698	315.32	1574.24	1529.07	3103.31
	120	0.0729	329.75	1646.32	1597.08	3243.40
C_h	8	0.0743	336.76	1345.04	1304.02	2649.06
	9	0.0702	316.96	1424.16	1383.10	2807.26
	10	0.0667	300.25	1498.99	1457.89	2956.89
	11	0.0636	285.92	1570.16	1529.04	3099.20
	12	0.0610	273.45	1638.17	1597.02	3235.18
a	4050	0.0682	293.71	1466.21	1423.22	2889.44
	4150	0.0674	297.00	1482.68	1440.66	2923.35
	4250	0.0667	300.25	1498.99	1457.89	2956.89
	4350	0.0660	303.48	1515.13	1474.93	2990.06
	4450	0.0653	306.67	1531.12	1491.77	3022.89
b	2460	0.0668	299.98	1496.06	1457.82	2953.91
	2560	0.0667	300.11	1497.54	1457.85	2955.40
	2660	0.0667	300.25	1498.99	1457.89	2956.89
	2760	0.0666	300.39	1500.43	1457.94	2958.37
	2860	0.0665	300.54	1501.86	1457.98	2959.85

Observations: From the table 3, it is observed that

1. With the increase in setup cost per unit (C_0), optimum quantity (Q*), Cycle time (T), Setup cost, Holding cost and total cost increases.
2. With the increase in holding cost per unit (C_h), optimum quantity (Q*), setup cost, Holding cost and cycle time (T) decreases but total cost increases.
3. Similarly, other parameters "a", "b" can also be observed from the table.

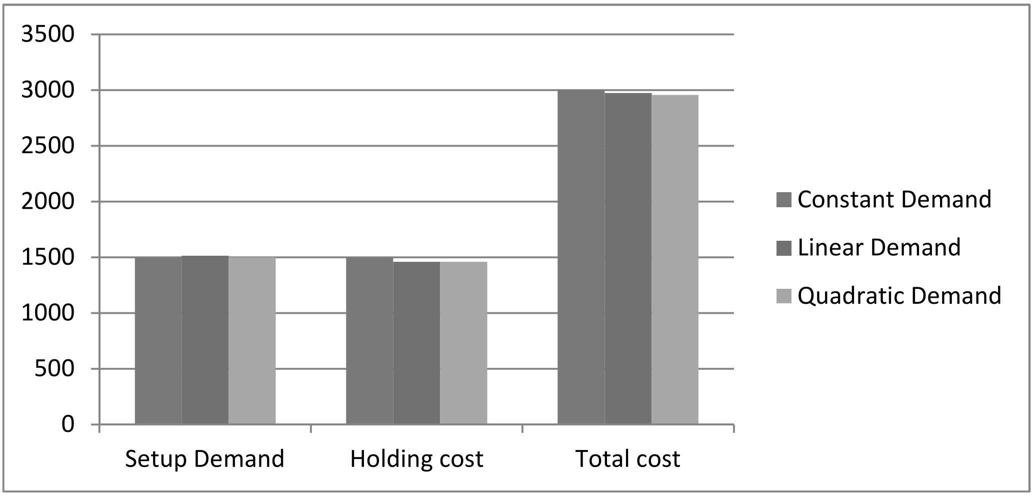

Figure 2: Comparison of holding cost between constant and continuous functions

Table 4: Comparisons of cost reduction in holding cost and total cost

Nature of cost	Constant demand	Linear demand	Quadratic demand
Setup cost	1500.00	1513.90	1498.99
Holding cost	1500.00	1458.78	1457.89
Total cost	3000.00	2972.68	2956.89

From the above study, a comparative study is carried out between constant, linear and quadratic demand. It is ascertained that holding cost in continuous functions (linear and quadratic functions) is less than the constant holding cost. Therefore, holding cost in linear and quadratic functions is better to compare to the holding cost in constant. Similarly, total cost in continuous functions (linear and quadratic functions) is less than the constant total cost. Therefore, the total cost in linear and quadratic functions is better to compare to the total cost in constant demand.

4. Conclusion

The objective of this paper is to develop a purchasing inventory models for obtaining optimum cycle time by using the time value of money approach in third and fourth order equations for constant, linear and quadratic demand. Several papers were published with constant, linearly increasing or decreasing demand and few papers were published with quadratic demand. Also, most works on inventory models do not take the optimal solution in higher order equation. In this paper, a purchasing inventory models for non-deteriorative items with constant, linear and quadratic demand with time value of money is considered and the optimum solution is derived in higher order equation. Three models are developed. In the first model, the purchasing inventory model with constant demand, in the second model, linear demand and in the third model, quadratic demand for non-deteriorative items is considered. A mathematical model is developed for each case and the optimal production lot size which minimizes the total cost is derived. The optimal solution is derived and an illustrative example is provided. The validation of result in this model was coded in Microsoft Visual Basic 6.0.

This research can be extended as follows:

a) Most of the production systems today are multi-stage systems and in a multi-stage system the defective items and scrap can be produced in each stage. Again, the defectives and scrap proportion for a multi-stage system can be different in different stages. Taking these factors into consideration this research can be extended for a multi-stage production process.

b) Traditionally, inspection procedures incurring cost is an important factor to identify the defectives and scrap and remove them for the finished goods inventory. For better production, the placement and effectiveness of inspection procedures are required which is ignored for this research, so inspection cost can be included in developing the future models.

c) The demand of a product may decrease with time owing to the introduction of a new product which is either technically superior or more attractive and cheaper than the old one. On the other hand, the demand of new product will increase. Thus, demand rate can be varied with time, so variable demand rate can be used to develop the model.

The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

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