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Optimality of the Bottleneck Product Rate Variation Problem with Asymmetric Objective Function

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Abstract

The bottleneck product rate variation problem minimizes the maximum deviation between the actual and the ideal cumulative production of a variety of models of a common base product. It is known as a sequencing problem in mixed-model just-in-time production systems. A common assumption is that early and tardy production of a product are equally undesired. Therefore, the problem has been extensively studied with the assumption of equal penalty to be issued for inventory and for shortage with several pseudo-polynomial exact algorithms and heuristics. However, there are cases where manufacturers judge earliness and tardiness differently, e.g. if failing to satisfy a customer is worse than keeping items on stock.

In this paper, we consider the problem with different penalties for inventory and for shortage and propose a pseudo-polynomial exact solution procedure.

Keywords: Product rate variation problem, Sequencing, Non-linear integer programming problem

1. Introduction

Manufacturing systems have developed from mass production to mass customization to satisfy the customer demands for a variety of high quality products at reasonable prices. Therefore, manufacturing companies strive to improve their sales by increasing the product variety with a balance strategy [25]. This balance strategy aims to minimize the complexity of a general mixed-model manufacturing system. It has recently been investigated and a solution procedure has been proposed by Wang et al. [26]. The opposed constraints of quality and costs force manufacturing companies to look for approaches to cut down costs without lowering down the quality of the products and services in the manufacturing process. For example, the Toyota Production System [20, 22], commonly known as the just-in-time production in manufacturing systems, aims to align the sequence of manufactured model variants to the actual demand of the customers in order to eliminate inventories. The challenge of this approach is to prevent shortages while keeping the inventory at a minimum level. Empirical observations have confirmed the positive effects of the implementation of just-in-time systems in manufacturing companies [see, e.g., 10].

Problem with symmetric objective function

Mixed-model just-in-time (abbreviated as MMJIT) production systems with negligible change-over costs between the models of a common base product have been extensively studied and implemented in order to respond to the customer demands for a variety of models without holding large inventories or incurring large shortages [20, 19,2,6]. The sequence planning has been an important issue in mixed-model assembly lines and is closely linked to the complexity of the system [28]. A sequence planning of different models of a common base product by keeping the rate of usage of all parts used by the assembly lines as constant as possible can be useful for the effective utilization of the systems. However, such an ideal production may not be possible during production. There exists a deviation between the exact cumulative productions and the ideal one. The problem which minimizes either the maximum or the total deviations between the actual cumulative productions over the observed periods from the ideal one is called the product rate variation problem (abbreviated as PRVP) [19].

The PRVP has been widely investigated as a non-linear integer programming problem, under the assumptions of negligible change-over cost, unit processing time, mixed demand, equal penalty for the inventory and for the shortage, and sufficient capacity of production since it has a model with a strong mathematical base and wide real-world applications, see [2,6]. Several heuristics with good performance have been developed though myopic characteristics occurred in them could not eliminated [24]. An exact algorithm with pseudo-polynomial time for the PRVP with the objective of minimizing the total deviations i.e. the total PRVP (abbreviated as TPRVP) has been investigated in [15]. The algorithm solves the TPRVP transforming into an equivalent assignment problem.

The bottleneck PRVP (abbreviated as BPRVP) i.e. the PRVP with the objective of minimizing the maximum deviation has also been solved with an exact solution procedure for its absolute deviation objective function [23]. The problem is transformed into a perfect matching problem which yields a feasible solution and a bisection search algorithm works for optimality. The solution procedure has been improved with an improved upper bound and with an investigation of the necessary and sufficient condition for the existence of a feasible solution [3]. However, the problem remained open for other objective functions since it required a suitable upper bound for the corresponding objective [5]. The PRVP with the objective function other than the absolute deviation has been solved in [7,11] by investigating the upper bounds corresponding to the objective functions.

The problem has some limitations for the effective implementation in the assembly lines [2] which may occur because of the assumptions for the modeling of the problem. It is interesting and important to relax some assumptions to find solution so that it could be implemented more effectively. Such attempts have been taken to some aspects. For example, the BPRVP with significant change-over cost and arbitrary processing time, instead of negligible changeover cost and unit processing time, has been well studied [see 27, 16]. This is a two phase multi-objective problem. The problem determines small batches of copies of models [see 27, 16] and then sequences the batches over the period of production [13].

In this paper, we relax another assumption of equal penalty to be issued for the earliness and for the tardiness and consider more penalty for the shortage than for the inventory since the tardiness seems to be more serious for the manufacturing companies in the competitive environment.

Earliness and Tardiness Penalties

In general, the problem of matching a specific due date for each job in scheduling decisions has been recognized as an important factor [8, 9]. The introduction of penalties for earliness in addition to tardiness made scheduling models more realistic in the light of the just-in-time philosophy, which not only requires products to be delivered without delay to the customer, but also to avoid inventory costs induced by early completion [1, 17]. This innovation has been included in nearly all scheduling approaches by now. Nevertheless, the vast majority of the scheduling models consider a quadratic performance measure, namely the sum of squared deviations of the completion times of the jobs from their due dates [4]. Cai and Zhou hypothesize that this is since quadratic performance measures are relatively easy to investigate if randomness is involved [4]. In their own work, they penalize earliness and tardiness with different weights in order to include the more realistic assumption that costs incurred by either earliness or tardiness is often different in practice.

In this paper, we investigate a solution procedure for the BPRVP with unequal penalty for the earliness and for the tardiness. More precisely, we assume that the penalty for tardiness is higher than the penalty for earliness, because frequent and extensive shortages are likely to threaten the trust of the customers in the ability of a company to deliver. Consequently, there is not only cost incurred by the actual late delivery, such as contractual penalties, but also opportunity by lost future demand if customers switch to competitors [21]. In turn, inventory cost is rather linear, with a slight exponential effect caused by obsolescence [18].

The paper is structured as follows. In Section 2, we present a non-linear integer programming formulation. Section 3 discusses the solution procedure for the problem with time window in Subsection 3.1, the feasible solution in Subsection 3.2, and optimality in Subsection 3.3. The last section concludes the paper.

2. Problem Formulation

Let the given time horizon be partitioned into D equal time units, where D stands for the total demand of n, $n \geq 2$ different models with d_i copies of model i with $i = 1, 2, \ldots, n$.

A copy of a model is produced in a time unit k, which means that the copy of the model is produced during the time period from k-1to k, with k = 1, 2, …, D. Let x_{ik} and kr_i, i = 1, 2, …, n; k = 1,2,…D, be the actual and the ideal cumulative productions, respectively, of model i produced during the time units 1 through k with $r_i = \frac{d_i}{D}$ to be the demand rate. An inventory holds if $x_{ik} - kr_i > 0$, and a shortage incurs if $kr_i - x_{ik} -> 0$. In this approach, we argue that it is reasonable to issue more penalty to the shortage than to the inventory (see Section 1). The modified non-linear integer programming for the problem is the following.

$$\text{Minimize} \quad F_{m_I m_S} = \max_{ik} \begin{cases} (x_{ik} - kr_i)^{m_I} \text{ for } x_{ik} - kr_i > 0 \\ (kr_i - x_{ik})^{m_S} \text{ for } kr_i - x_{ik} > 0 \end{cases} \qquad (1)$$

where m_S, m_I with $m_S < m_I$ are positive integers

subject to

$$\sum_{i=1}^{n} x_{ik} = k, \qquad k = 1,2,\ldots,D \qquad (2)$$

$$x_{i(k-1)} - x_{ik}, \; i = 1, 2, \ldots n; \; k = 2, 3, \ldots, D \qquad (3)$$

$$x_{iD} = d_i, x_{i0} = 0, \; i = 1, 2, \ldots, n \qquad (4)$$

$$x_{ik} \geq 0, \text{ integer}, \; i = 1, 2, \ldots, n; \; k = 1, 2, \ldots, D \qquad (5)$$

3. Solution Procedure

The BPRVP with symmetric objective function has been solved with an exact solution method in pseudo-polynomial time. The method is based on perfect matching extracted from a convex bipartite graph for the feasible solution and a bisection search for the optimality. The BPRVP with asymmetric objective function can also be solved with this procedure with necessary modifications. The convex bipartite graph is constructed over the time period, say time window, where a copy of a model can be sequenced.

3.1. Time Window

The time window of a copy of a model i, i = 1, 2, …, n, is determined from the integral points where a suitable upper bound B > 0 of the function value crosses the level curves i.e. the curves obtained from the corresponding bijective function. The j^{th} copy of model i with i = 1, 2, …, n; j = 1, 2, …d_i, i.e. (i, j), is sequenced in a time unit k in the time horizon [1,D] such that the level curves do not exceed B. There exist nD deviations between the actual and the ideal cumulative productions of D copies of n models with only n+D different values since the actual cumulative production x_{ik}, i = 1, 2, …, n; k = 1, 2,…,D, is sequence-dependent integer from $\{0,1,\ldots,d_i\}$. However, the value of the ideal cumulative production kr_i, i = 1, 2, …, n; k = 1, 2, …, D, is sequence independent rational number such that $kr_i \epsilon \left\{\frac{d_i}{D}, \frac{2d_i}{D}, \ldots, d_i\right\}$, i = 1,2,…,n.

The level curve for copy (i, j) of the objective function is defined as

$$\text{Minimize} \quad f_{ij}^{\,m_I m_S} = \max_{ik} \begin{cases} (j - kr_i)^{m_I} & \text{for } x_{ik} - kr_i > 0 \\ (kr_i - j)^{m_S} & \text{for } kr_i - x_{ik} > 0 \end{cases} \qquad (6)$$

Fig. 1 shows the level curves for an instance (4,6, 10) with unequal penalties.

The time window for copy (i, j) is $T_{m_I m_S}(i,j) = [Em_I\,(i,j), Lm_S\,(i,j)]$ (7)

where $Em_I\,(i,j)$ and $Lm_S\,(i,j)$ are the earliest and the latest sequencing times, respectively. We can derive the time window for a copy (i, j), i = 1, 2, …, n; j = 1, 2, …, d_i.

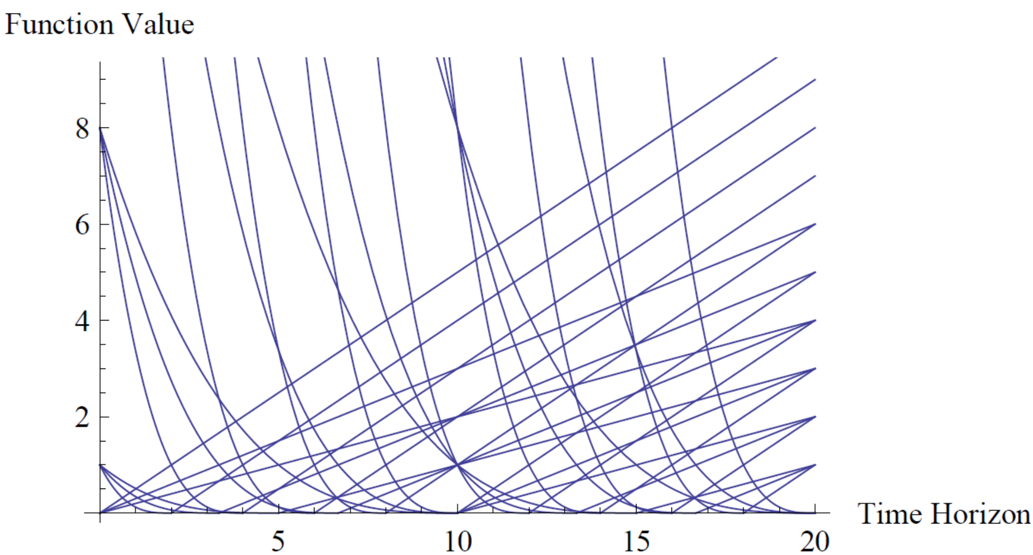

Figure 1. Level curves f^{31}_{ij} for the instance (d_1 = 4, d_2 = 6, d_3 = 10)

Theorem 1. Given a suitable upper bound B, the time window $T_{m_I m_S}(i,j)$ for a copy (i, j), i = 1, 2, …, n; j = 1, 2, …, d_i, is

$$T_{m_I m_S}(i,j) = \left[\left\lceil \frac{j - \sqrt[m_I]{B}}{r_i} \right\rceil, \left\lfloor \frac{j - 1 + \sqrt[m_S]{B}}{r_i} \right\rfloor\right] \qquad (8)$$

Proof:

The earliest sequencing time $Em_I\,(i,j)$ is the unique integer in [1,D] such that when (i, j) is sequenced at $Em_I\,(i,j) - 1$, the level curves exceed B but do not exceed when sequenced at $Em_I\,(i,j)$. The two inequalities $Em_I\,(i,j)$ satisfies are

$$(j-(Em_I\,(i,j)-1)r_i)^{m_I} > B \tag{9}$$

and

$$(j-Em_I\,(i,j)r_i)^{m_I} \le B \tag{10}$$

i.e. $$\frac{j-\sqrt[m_I]{B}}{r_i} \le Em_I\,(i,j) < \frac{j-\sqrt[m_I]{B}}{r_i}+1 \tag{11}$$

Therefore,

$$Em_I\,(i,j) = \left\lceil \frac{j-\sqrt[m_I]{B}}{r_i} \right\rceil \tag{12}$$

holds.

Likewise, the latest sequencing time $Lm_S\,(i,j)$ is the unique integer in [1,D] such that when (i, (j -1)) is sequenced at $Lm_S\,(i,j)-1$, the level curves do not exceed B but exceed when sequenced at $Lm_S\,(i,j)$. Then $Lm_S\,(i,j)$ satisfies the following two inequalities

$$[(Lm_S\,(i,j)-1)r_i-(j-1)]^{m_s} \le B \tag{13}$$

and

$$[Lm_S\,(i,j)r_i-(j-1)]^{m_s} > B \tag{14}$$

i.e.

$$\frac{j-1+\sqrt[m_S]{B}}{r_i} < Lm_S\,(i,j) \le \frac{j-1+\sqrt[m_S]{B}}{r_i}+1 \tag{15}$$

Thus, $Lm_S\,(i,j) = \left\lfloor \frac{j-1+\sqrt[m_S]{B}}{r_i} \right\rfloor$ holds.

One may consider additional importance to some models by issuing a weight w_i for a model i, i = 1, 2, …, n.

The time windows for each copy of each model of an instance (4, 6, 10) at an upper bound $B = \frac{1}{2}$

are as follows. The time windows for the four copies of the first model of the instance are [2, 3], [7, 8], [12, 13] and [17, 18], respectively. Similarly, for the second models the time windows are [1, 2], [5, 6], [8, 9], [11, 12], [15, 16] and [18, 19], respectively and that for the third models [1, 2], [3, 4], [5, 6], [7, 8], [9, 10], [11, 12], [13, 14], [15, 16], [17, 18] and [19, 20], respectively.

The upper bound B > 0 has been investigated for the BPRVP with symmetric objective function. The upper bound for the problem with absolute deviation objective was investigated to be B = 1 [23]. It was then improved with the value $B = 1 - \frac{1}{D}$, [3]. The value has been modified to be $B = (1 - \frac{1}{D})^m$, m being a positive integer, for the problem with different objective functions [11]. The upper bound has been recently improved to be $B = (1 - r_{min})^m$ where r_{min} is the minimum demand rate [see 12]. The upper bound for the problem with assymetric objective can be modified to be

$$B = \begin{cases} (1 - r_{min})^{m_I} \text{ for } x_{ik} - kr_i > 0 \\ (1 - r_{min})^{m_S} \text{ for } kr_i - x_{ik} > 0^- \end{cases} \quad (16)$$

For a given upper bound B, the time window $T_{m_I m_S}(i, j)$ can be calculated for each (i, j), i = 1, …, n; j = 1, …, d_i, in time O(D).

3.2. Feasible Solution

We seek a feasible solution extracting a perfect matching from a convex bipartite graph. A convex bipartite graph $G = (V_1 \cup V_2, \varepsilon)$, with $V_1 = \{1, 2, \dots, D\}$; $V_2 = \{(i, j) | i = 1, \dots, n;\ j = 1, \dots, d_i\}$ and $\varepsilon = \{(k, (i, j)) |\ k \epsilon T_{m_I m_S}(i, j)\}$ is constructed by sequencing each copy (i, j), i = 1, 2, …, n; j = 1, 2, …, d_i, with each time unit within the time window $T_{m_I m_S}(i, j)$. The time window is feasible if a perfect matching can be extracted from the graph G. A perfect matching in G with $|V_1| = |V_2|$exists if and only if $|N(K)| \geq K$ for all $K \subseteq V_1$ and $N(K) = \{(i, j): (i, j) \epsilon V_2, \exists\, k \epsilon K \text{ s.t. } (k, (i, j) \epsilon E)\}$ [3]. This is the modified Hall's theorem in which an upper bound B > 0 must satisfy the two inequalities given in the following theorem.

Theorem 2. Given a convex bipartite graph G. There exists at least a perfect matching if and only if, for all $k_1, k_2 \epsilon V_1$ with $k_1 \leq k_2$ and $[E_{m_I}(i, j, L_{m_S}(i, j)] \cap [k_1, k_2] \neq \emptyset$

B satisfies the inequalities

$$\sum_{i=1}^{n} \left(\left\lfloor k_2 r_i + \sqrt[m_I]{B} \right\rfloor - \left\lceil (k_1 - 1) r_i - \sqrt[m_S]{B} \right\rceil \right) \geq k_2 - k_1 + 1 \quad (17)$$

$$\sum_{i=1}^{n} \left(\left\lceil k_2 r_i - \sqrt[m_I]{B} \right\rceil - \left\lfloor (k_1 - 1) r_i + \sqrt[m_S]{B} \right\rfloor \right) \leq k_2 - k_1 + 1 \quad (18)$$

Proof:

We show that the two inequalities are the consequences of Hall's condition $|N(K)| \geq K$ for all

$K \subseteq V_1$ where $N(K) = \{(i, j): (i, j) \epsilon V_2, \exists\, k \epsilon K \text{ s.t. } (k, (i, j) \epsilon E)\}$.

Let $K = [k_1, k_2] \subseteq V_1$ with $k_1 \leq k_2$. Then $(i, j) \in N(K)$.

Since $[E_{m_I}(i,j, L_{m_S}(i,j)] \cap [k_1,k_2] \neq \emptyset$

we can write $E_{m_I}(i,j) \leq k_2$ and $L_{m_S}(i,j) \geq k_1$

That is $\frac{j-\sqrt[m_I]{B}}{r_i} \leq k_2$ and $\frac{j-1+\sqrt[m_S]{B}}{r_i} \geq k_1$

So

$$\left\lceil (k_1-1)r_i + 1 - \sqrt[m_S]{B} \right\rceil \leq j \leq \left\lfloor k_2 r_i + \sqrt[m_I]{B} \right\rfloor$$

Therefore, for $K = [k_1,k_2] \subseteq V_1, |N(K)| \geq K$ if and only if

$$\sum_{i=1}^{n}\left(\left\lfloor k_2 r_i + \sqrt[m_I]{B}\right\rfloor - \left\lceil (k_1-1)r_i - \sqrt[m_S]{B}\right\rceil\right) \geq k_2 - k_1 + 1$$

Now consider

$$[E_{m_I}(i,j), L_{m_S}(i,j)] \subseteq k_1,k_2] \subseteq V_1$$

Then

$$k_1 \leq E_{m_I}(i,j) \text{ and } L_{m_S}(i,j) \leq k_2$$

That is

$$k_1 \leq \frac{j-\sqrt[m_I]{B}}{r_i} \text{ and } \frac{j-1+\sqrt[m_S]{B}}{r_i} + 1 \leq k_2$$

Which implies

$$\left\lfloor (k_1-1)r_i + 1 + \sqrt[m_I]{B} \right\rfloor \leq j \leq \left\lceil k_2 r_i - \sqrt[m_S]{B} \right\rceil$$

Thus, for $N(K) = [k_1,k_2] \subseteq V_1, |N(K)| \geq K$ if and only if

$$\sum_{i=1}^{n}\left(\left\lceil k_2 r_i - \sqrt[m_I]{B}\right\rceil - \left\lfloor (k_1-1)r_i + \sqrt[m_S]{B}\right\rfloor\right) \leq k_2 - k_1 + 1.$$

Hence the proof.

A perfect matching can be extracted from the graph G with the earliest due date (EDD) rule [23]. The EDD rule matches each time unit $k \in V_1$ to the unmatched copy (i, j) with the smallest $L_{m_S}(i,j)$ and $(k,(i,j)) \in \varepsilon$ to find a perfect matching. This is a modified Glover's EDD rule in which if an unmatched (i, j) does not exist for any k, the algorithm stops instead of moving to k + 1. The modified EDD algorithm sequences the lower numbered copies of a model to earlier sequencing times than the higher numbered copies, which leads the perfect matching extracted from G to be order-preserving.

Lemma 1. Let $\tilde{\varepsilon}$ be a perfect matching extracted from G. The perfect matching preserves the order.

Proof:

Let $\tilde{\varepsilon}$ be a perfect matching extracted from G using the EDD rule. Since the EDD rule matches each $k \in V_1$ to the unmatched (i,j) with the smallest $L_{m_S}(i,j)$, it suffices to show that $L_{m_S}(i,j)$ preserves the order.

Clearly $0 < r_i < 1$, i = 1, …, n,

$$L_{m_S}(i,j) = \left\lfloor \frac{j-1+\sqrt[m_S]{B}}{r_i} \right\rfloor < \left\lfloor \frac{j-1+\sqrt[m_S]{B}}{r_i} + 1 + \frac{1}{r_i} \right\rfloor = L_{m_S}(i,j+1).$$

The following theorem assures that there exists an order-preserving perfect matching in G if and only if there exists a feasible solution to the problem.

Theorem 3. Given a convex bipartite graph G with an upper bound B > 0. There exists a feasible solution to the problem if and only if there exists an order-preserving perfect matching in G.

Proof:

Consider a feasible sequence s for any instance to the problem. Then every (i, j), i = 1, …, n; j = 1, …, d_i, is issued to a unique time unit k, k = 1, …, D. Since the total number of copies of all models is exactly equal to the number of time units of the time horizon, there is a bijection $(i,j) \rightarrow k$, where $k \in V_1$ and $(i,j) \in V_2$, i = 1, …, n; j =1, …, d_i. The bijection yields a perfect matching. Lemma 1 shows that the perfect matching is order-preserving.

Conversely, suppose that $\tilde{\varepsilon} \subseteq \varepsilon$ be an order-preserving perfect matching in G. If $\big(k,(i,j_1)\big)$, $(k,(i,j_2)) \in \tilde{\varepsilon}$. then $(i,j_1) = (i,j_2)$This shows that no two copies compete for the same time unit. Moreover, since $\tilde{\varepsilon}$ is perfect, there remains no time unit $\tilde{k} \in V_1$ unmatched. The order-preserving of the perfect matching yields a feasible sequence.

The sequences 3-2-1-3-3-2-3-1-2-3-3-2-1-3-3-2-3-1-2-3 and 2-3-1-3-2-3-1-3-2-3-2-3-1-3-2-3-1-3-2-3 are the two feasible production sequences which are obtained using the above mentioned procedure.

3.3. Optimality

If an instance with the demands, (d_1, d_2, …, d_n), has a feasible sequence with an objective function value equal to the lower bound, this sequence is optimal. The above mentioned feasible production sequences are optimal since they are obtained using the time window extracted at the lower bound $(1-r_{max})^{m_I}$ for the inventory and $(1-r_{max})^{m_S}$ for the shortage. However, such a sequence does not exist in many instances [14]. There always exists at least a feasible sequence with an objective function value equal to the upper bound in all instances. Let

$$\mathbb{x} = \{x = (x_{ij}) | \, i = 1, \dots, n; j = 1, \dots, d_i\}$$

be the set of all feasible solutions. A sequence $\tilde{x} \in \mathbb{x}$ with an objective function value equal to the minimum upper bound is optimal. A bisection search algorithm that runs in the interval between the lower and the upper bounds finds the minimum upper bound for a particular instance. The algorithm yields an optimal sequence in O(logD) time [23]. The interval over which the bisection search runs for the BPRVP with the asymmetric objective function is

$$[(1-r_{max})^{m_S}, (1-r_{min})^{m_I}].$$

4. Concluding remarks

The product rate variation problem with equal penalty for the inventory and for the shortage has been extensively investigated with a number of heuristics and exact solution approaches which are solvable in polynomial time based on the size of the instance. In this paper, the bottleneck case of the problem with different penalty for the inventory and for the shortage has been studied with an exact solution procedure to obtain an optimal solution with the same complexity of the previous problem. A comparative study between the optimal solutions for the problems with equal and unequal penalty will be a logical next step for further research, so that the impact of differences in the preferences towards inventory or shortages can be made explicit.

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